Optimum Design of a Tee Mixer for Fast Reactions

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Turbulence promotes most important chemical reactions, heat transfer operations, and mixing and combustion processes in industry. Effective use of turbulence increases reactant contact and decreases reaction times, which can significantly reduce the cost of producing many chemicals. Efficient mixing is necessary to obtain profitable yields or to eliminate excessive corrosion in reactor or combustion chambers.

If necessary contact time between fluids is short, it is common in many existing chemical process units to continuously mix two fluids in a pipeline with subsequent transport to other locations. Although the continuous mixing of two fluid streams can be achieved using a number of mixer geometries, many procedures such as the use of baffles or complex internal geometries will introduce excessive pressure drops and significantly increase the cost of the mixing device. An effective, simple method to mix two fluids within a pipeline is to introduce feed jets such that mixing occurs rapidly within the turbulent core of the pipeline.

Frequently, a pipeline tee mixer must be designed for a special processing requirement. Examples of such requirements include the rapid mixing of a small fluid stream into a pipeline of limited length such that flashing or the formation of undesirable products is avoided. Other requirements may be a desired degree of uniformity of the mixture for a specified mixing ratio or mixer geometries that minimize corrosion, scaling, or thermal shock to the walls. Additional information is available in the reviews of pipeline mixing with tees and other geometries prepared by Simpson (1974), Gray (1986), and Forney (1986).

Ger and Holley (1976) and Fitzgerald and Holley (1981) compared the standard deviation of measured tracer concentrations far downstream (7-120 pipe diameters) from the side tee. Although the objective of the research, in both the near and far field, was to establish optimum conditions for a pipeline tee mixer, the experimental data were limited and the results were inconclusive. Typically, the standard deviation or second moment of the tracer concentration was observed to decrease with increasing jet momentum at a fixed measurement point downstream. However, it was difficult to establish a distinct minimum in the second moment of the tracer concentration distribution with increasing jet momentum, particularly within the first 20 pipe diameters from the injection point.

The mixing criteria used in many of the experiments assumed that optimum mixing in a pipeline was achieved if the jet was centered along the pipeline axis (Forney et al., 1979, 1982, 1985). The assumption of a geometrically centered jet appeared to be useful if the measurement point was at distances far from the injection point or $15 < x/D < 120$. More recently, Sroka and Forney (1989) provided a mathematical basis for the prediction of concentration second moments for the first 15 pipe diameters downstream from the injection point. The latter results indicate that the second moment of the tracer concentration decreases with increasing jet momentum and distance from the injection point. The simple scaling law developed by Sroka and Forney appeared to correlate the data of Holley et al. (1976, 1981) and Maruyama et al. (1981, 1983).

It may be desirable, however, to promote rapid mixing of two fluids with a tee mixer in a short distance downstream from the injection point at $x/D < 3$. In particular, the suitability of pipeline mixing tees for reactor applications, where the reaction times are small, depends on achieving homogeneity of the reactant concentrations in short times. Tosun (1987) studied the product yield of tee mixers with competitive consecutive reactions. The experimental data of Tosun demonstrated a distinct minimum in the undesirable product yield for certain tee mixer geometries. Cozewith and Busko (1989) measured the distance downstream from the tee inlet required for the neutralization of a base indicator. Cozewith and Busko found a minimum distance to mix for certain tee mixer geometries.

The experimental work of Cozewith and Busko (1989) demonstrated that it is necessary to increase the momentum of the side tee such that the secondary fluid impinges on the opposite wall of the pipe near the tee inlet. Some of the data of Maruyama et al. (1983) and Gosman and Simitovic (1986) also indicated that mixing of an inert tracer could be improved by the impingement of the secondary fluid against the opposite wall of the pipe near the tee inlet. Moreover, recent numerical calculations by Cozewith et al. (1990) with a k–ϵ turbulent model also support the idea that pipeline impingement is necessary to minimize the second moment of a tracer concentration for a fixed tee mixer geometry near the tee inlet $x/D < 3$.

Theory

We assume here that the fluids to be mixed are of the same phase and are miscible. We also assume that fully developed turbulent flow exists in both the pipeline and tee inlet such that the Reynolds number of either flow has no effect on the quality of mixing. Experimental measurements indicate that the latter requirement is satisfied if the Reynolds number of the pipe $Re_p > 10{,}000$ and that of the jet $Re_j > 6{,}000$ (O'Leary and Forney, 1985; Cozewith and Busko, 1989).

For the case of pipeline impingement, we hypothesize that the process is dynamically similar if the secondary fluid or jet reaches the opposite wall of the pipe such that the concentration of an inert tracer is fixed relative to its completely mixed or bulk mean value. Consider a tracer concentration c_o at the tee inlet as shown in Figure 1. The tracer is completely mixed when the tracer achieves a minimum concentration or bulk mean value of

$$\frac{\bar{c}}{c_o} = \frac{q}{q + Q} \tag{1}$$

where $q\,(=d^2 u_o)$ and $Q\,(=D^2 v)$ are the volume flow rates for the tee inlet and pipe, respectively. Thus, we assume that the concentration of the jet at the point of impingement where $x = L$ and $z = D$ is constrained such that

$$\frac{c_L/c_o - \bar{c}/c_o}{\bar{c}/c_o} = \epsilon. \tag{2}$$

Here, ϵ is a constant limited by the values $0 < \epsilon < (D/d)^2(1/R)$ where, $R\,(=u_o/v)$ is the velocity ratio and c_L is the maximum jet tracer concentration at the point of impingement.

Dimensional arguments for a jet in the near field suggest that the maximum jet concentration varies with distance z such that

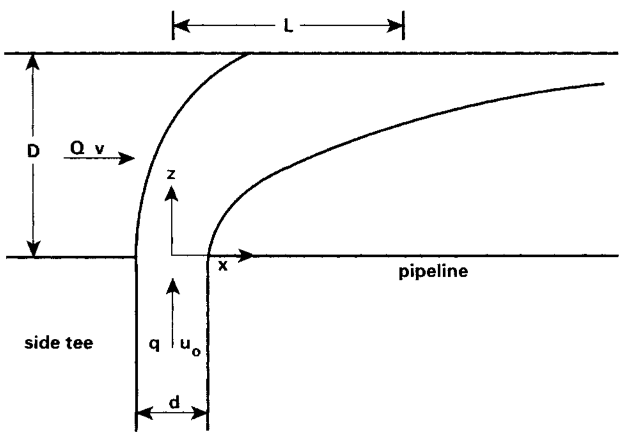

Figure 1. Tee mixer.

(Wright, 1977)

$$\frac{c}{c_o} = \frac{kq}{du_o z}, \quad R(d/D) > 1 \tag{3}$$

where $k = 2.1$. At the point of impingement where $z = D$ and $x = L$, one obtains from Eq. 3

$$\frac{c_L}{c_o} = k\left(\frac{d}{D}\right). \tag{4}$$

Substituting Eqs. 1 and 4 into the constraint of Eq. 2, one obtains

$$\epsilon + 1 = k\left(\frac{d}{D}\right)(1 + Q/q)$$

or

$$\beta = \left(\frac{d}{D}\right)\left[1 + \left(\frac{D}{d}\right)^2\left(\frac{1}{R}\right)\right] \tag{5}$$

where $\beta = (\epsilon + 1)/k$. Solution of Eq. 5 yields a velocity ratio for optimum mixing

$$R = \frac{1}{d/D(\beta - d/D)} \tag{6}$$

where the constant β must have values in the range

$$1/k < \beta < \left[1 + \left(\frac{D}{d}\right)^2\left(\frac{1}{R}\right)\right]\left(\frac{1}{k}\right). \tag{7}$$

The distance downstream from the tee inlet where the jet impinges against the opposite pipe wall for the optimum velocity ratio can be obtained from the expression for the jet trajectory (Wright, 1977)

$$\frac{z}{dR} = f(R)\left(\frac{x}{dR}\right)^{1/2}, \quad R\left(\frac{d}{D}\right) > 1 \tag{8}$$

where the function (Forney and Lee, 1982)

$$f(R) = \frac{0.17}{0.1 + 0.35/R^{1.25}}. \tag{9}$$

Substituting $z = D$ and $x = L$ into Eq. 8, one obtains the distance to impingement

$$\frac{L}{D} = \frac{1}{f^2(R)}\left(\frac{D}{d}\right)\left(\frac{1}{R}\right) \tag{10}$$

where R represents the optimum jet-to-pipe velocity ratio and is expressed in terms of the diameter ratio d/D of the tee mixer from Eq. 6.

Results and Discussion

A least square curve fit was made from 12 data points with Eq. 6 and the result is plotted in Figure 2. The data from the

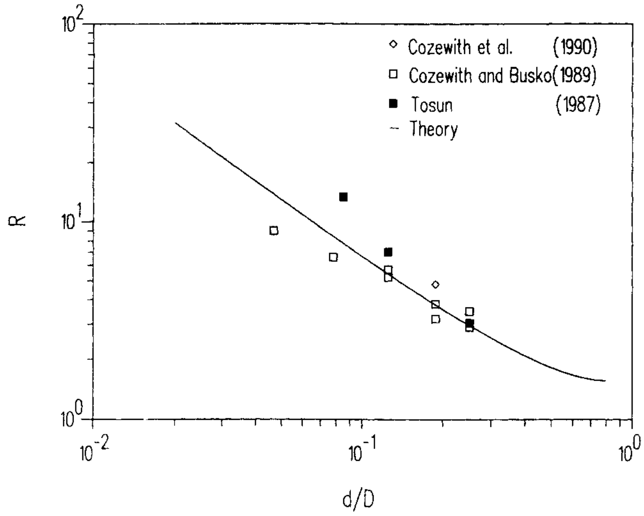

Figure 2. Optimum velocity ratio vs. diameter ratio (β = 1.6).

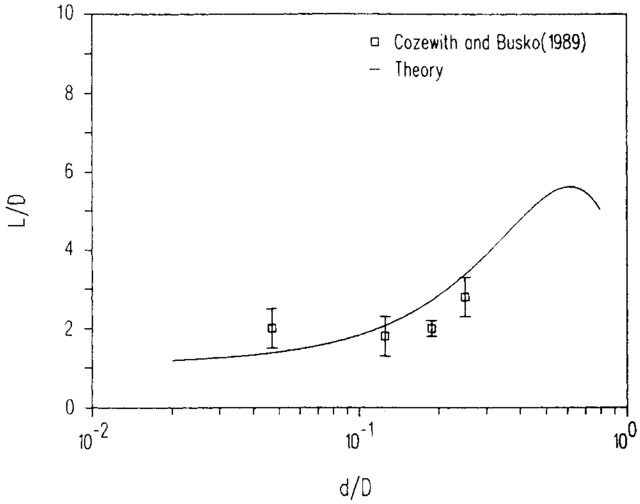

Figure 3. Required pipe length for impingement at the optimum velocity ratio.

Ratio of side tee inlet concentration to stoichiometic is α = 0.26 for experimental data points.

work of Tosun (1987), Cozewith and Busko (1989), and Cozewith et al. (1990) represent optimum velocity ratios for the indicated jet-to-pipe diameter ratios covering the range $0.047 \leq d/D \leq 0.25$. All of the data points, with the exception of two for $d/D < 0.1$, were found to be correlated with Eq. 6 to within 20% for a value of the constant $\beta = 1.6$. This yields a value of $\epsilon = 2.36$ in Eq. 2, which represents the fraction of the original concentration of an inert tracer left to mix at the point of impingement. The value of $\beta = 1.6$ also lies within the indicated range of Eq. 7 for all the values of the velocity and diameter ratios.

It is interesting to note that Eq. 6 correlates all of the available data without regard to the stoichiometry of the reaction or the reaction rates. The data of Cozewith and Busko (1989) represent a diffusion-limited acid-base reaction, while the data of Tosun (1987) represent an intermediate case where diffusion and reaction rates are comparable. The remaining computational data point of Cozewith et al. (1990) was determined by minimizing the standard deviation of an inert tracer. Apparently, similarity in macromixing with a tee mixer assures similarity in micromixing.

There is one remaining quantity of interest for a tee mixer that operates with sufficiently large jet momentum such that impingement of the jet occurs. This is the distance downstream $x = L$ at which the jet contacts the wall. Equation 10, which represents the distance to impingement, is plotted in Figure 3. As indicated, L/D increases from a minimum of roughly one for small d/D ratios to a maximum of approximately six for large d/D.

The smaller limit of $L/D \rightarrow 1$ for small diameter ratios d/D as shown in Figure 3 was observed by Cozewith and Busko (1989) for all tee mixer diameter ratios in the limit of large acid concentrations relative to the base indicator. The data points plotted in Figure 3 were derived from the measurements of Cozewith and Busko (1989). In this case, the initial base concentration at the tee inlet was reduced to a fraction α of the required stoichiometric value or

$$\frac{c_{Bo}}{c_{Ao}} = \frac{\alpha}{R(d/D)^2} \tag{11}$$

where $\alpha = 0.26$. Values of the impingement length L/D were determined from measured contours of L/D vs. initial concentration ratios $1/[1 + \alpha/R(d/D)^2]$ for constant d/D and R from the work of Cozewith and Busko. The value of $\alpha = 0.26$ chosen implies that the acid and base concentrations were nearly equal to their stoichiometric value at the point of impingement, since $\alpha \simeq 1/(1 + \epsilon)$.

The solid curve plotted in Figure 3 was derived from Eq. 10 by substituting the optimum velocity ratio from Eq. 6. Since Eq. 10 does not represent an explicit expression for the impingement length in terms of the jet-to-pipe diameter d/D at the optimum velocity ratio R, a least square curve fit was made with a cubic polynomial. The polynomial shown below is accurate to within 3% of values plotted in Figure 3. Thus,

$$\frac{L}{D} = 0.992 + 7.77\left(\frac{d}{D}\right) + 11.5\left(\frac{d}{D}\right)^2 - 19.1\left(\frac{d}{D}\right)^3. \tag{12}$$

Notation

c = local maximum jet concentration, mol/m^3
d = diameter of side tee, m
D = diameter of pipeline, m
k = constant ($\simeq 2.1$)
L = distance to impingement point, m
q = volumetric flowrate in side tee ($=\pi d^2 u_o/4$), m^3/s
Q = volumetric flowrate in pipeline ($=\pi D^2 v/4$), m^3/s
R = velocity ratio ($=u_o/v$)
u_o = fluid velocity in side tee, m/s
v = fluid velocity in pipeline, m/s
x = distance downstream from side tee, m
z = distance across pipeline, m

Greek letters

α = ratio of side tee concentration to stoichiometric value
β = constant [$=(\epsilon + 1)/k$]
ϵ = concentration ratio ($=c_L/\bar{c} - 1$)

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Errata

In the paper "Prediction of Liquid Circulation in Viscous Bubble Columns" by R. G. Rice and N. W. Geary (**36**, September 1990, p. 1339), the following corrections are made:

In Eq. 46, the denominator of the integrand should read "$\psi(x)^2$," not "$\psi(x)^3$."

The bracket in Eq. 22 should read "$[1 - (\epsilon/\lambda)^m]$," i.e., the second equality sign should be replaced with unity.

The expression following Eq. 42 to compute bubble frequency should read:

$$f^{-1} = V_o^{1/6} \cdot V/(G\sqrt{g})$$